Two Higgs doublet model within USY hypohtesis

Katsuichi Higuchi[a] and Katsuji Yamamoto[b]
[a]Department of Literature, Kobe Kaisei College, Kobe 657-0805, Japan
[b]Department of Nuclear Engineering, Kyoto University, Kyoto 606-8501, Japan

Abstract

We investigate phenomenology in a Two Higgs Doublet Model (THDM) within the Universality Strength for Yukawa couplings (USY) hypothesis. It is assumed in this model that all the Yukawa couplings have the same moduli. The two Higgs doublets are also supposed to develop vacuum expectation values (vev's) with the same moduli. In favor of this mechanism, not only the quark mass hierarchical relations $m_u, m_c \ll m_t$, $m_d, m_s \ll m_b$ but also $m_b \ll m_t$ are naturally explained. Furthermore, the sufficiently large phase invariant CP violation parameter J is obtained. In addition, the scalar mediated FCNC's are generated at the tree level.

1. Introduction

At present, phenomena of particle physics are almost understood within the Standard Model (SM) [1]. However, there are many problems in the SM. Particularly, the flavor structure of the Yukawa couplings cannot be predicted, without providing a reasonable explanation for the quark mass hierarchy, $m_u, m_c \ll m_t$, $m_d, m_s \ll m_b$ and $m_b \ll m_t$. In order to solve this flavor problem, many suggestions have been proposed in literature. The Universality Strength for Yukawa couplings (USY) hypothesis was proposed in 1990 [2] as an interesting solution, and since then it has been studied extensively [2-12].

In the SM within USY hypothesis, while the six ordinary quark masses and the absolute values of the Cabbibo-Kobayashi-Maskawa matrix elements $|V_{CKM}|$ are reproduced correctly, the sufficiently large phase invariant CP violation parameter $J \equiv \mathrm{Im}(V_{im} V_{jn} V_{jm}^* V_{in}^*)$ cannot be obtained, as shown by Branco, et al. [4]. Then, in order to reproduce the experimental value of J, an extended USY model including extra quark singlets and Higgs singlets was studied[12].

It this article, a Two Higgs Doublet Model (THDM) within USY hypothesis is proposed as an alternative USY model. Two specific ansatz are considered.

- In view of the grand unified theory (GUT) the moduli of the Yukawa couplings for the up-type quarks are the same as those for the down-type quarks.
- The vev's of the two Higgs doublets take the same absolute value.

  Then, the following results will be obtained.
- The scalar mediated FCNC's appear at the tree level, while the Z mediated ones do not.
- The top-botom mass hierarcy $m_b \ll m_t$ is naturally explained. In addition, the CKM quark mixing matrix V_{CKM} and rephrasing invariant CP violation measure J can be reproduced.

This article is organized as follows. In Sec 2, the SM within USY hypothesis is reviewed. In Sec 3, the THDM within USY is presented, and its phenomenology is investigated. Sec 4 is devoted to summary.

2. Review of the SM within USY hypothesis

2-1. The democratic limit

It will be shown that the quark mixing between the first and second generations in V_{CKM} is obtained in the democratic limit.

A 3×3 democratic ordinary quark mass matrix is given by

$$M_\alpha = \frac{\lambda_\alpha v}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} . \tag{2.1}$$

where $\alpha = u$ or d. It is diagonalized as

$$M_\alpha \to F_\alpha{}^T M F_\alpha = M_{\alpha,diag} = \frac{\lambda_\alpha v}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \qquad (2.2)$$

where F_α is an orthogonal matrix composed of the three eigenvectors of M_α. Here, it is naturally seen that the quark masses of the third generation are much larger than those of the first and second generations. The eigenvalues and eigenvectors of M_α are shown in table 1.

Table1.The eigenvalues and eigenvectors of M_α.

eigenvalues	eigenvectors
$m_u = 0$	$\mathbf{x}_1 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$
$m_c = 0$	$\mathbf{x}_2 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$
$m_t = \frac{3\lambda_\alpha v}{\sqrt{2}}$	$\mathbf{x}_3 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Because of the two degenerate eigenvalues, $m_u = m_c = 0$, their eigenvectors may be arbitrary linear combinations of $\mathbf{x}_1$ and $\mathbf{x}_2$.

We start with the basis of the up-type quarks as

$$F_u = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}. \qquad (2.3)$$

The eigenvectors of M_d, $\mathbf{x}_1'$, $\mathbf{x}_2'$ and $\mathbf{x}_3'$ are given as

$$\begin{cases} \mathbf{x}_1' = \mathbf{x}_1 \cos\theta + \mathbf{x}_2 \sin\theta \\ \mathbf{x}_2' = \mathbf{x}_1 \sin\theta - \mathbf{x}_2 \cos\theta \\ \mathbf{x}_3' = \mathbf{x}_3 \end{cases}, \tag{2.4}$$

providing the basis of the down-type quarks,

$$F_d = \begin{bmatrix} \mathbf{x}_1' \\ \mathbf{x}_2' \\ \mathbf{x}_3' \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \cos\theta + \mathbf{x}_2 \sin\theta \\ \mathbf{x}_1 \sin\theta - \mathbf{x}_2 \cos\theta \\ \mathbf{x}_3 \end{bmatrix}, \tag{2.5}$$

where $\mathbf{x}_1'$, $\mathbf{x}_2'$ and $\mathbf{x}_3'$ and are orthogonal each other. The CKM matrix is given by

$$V_{CKM} = F_u F_d^{\dagger}, \tag{2.6}$$

which in democratic limit becomes as

$$V_{CKM} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.7}$$

By taking the input value $\theta = 0.227$ [rad], the experimental result of the quark mixing between the first and second generations are naturally reproduced in the democratic limit. It is, however, clear that the quark mixing involving the third generation cannot appear in Eq. (2.7) of the democratic limit.

2-2. Reproducing the experimental values

In the democratic limit, either quark masses of the first and second generations or the quark mixing involving the third generation are not generated. Then, keeping the advantage of flavor democracy, some extended scheme should be considered to generate them. The USY is a powerful mechanism in such an extended flavor democratic model. Within USY mechanism these can be generated by the aid of adding phases to the Yukawa couplings.

The quark mass matrices are provided within USY as

$$M_\alpha = \frac{\lambda_\alpha v}{\sqrt{2}} \begin{bmatrix} e^{i\phi_{\alpha 11}} & e^{i\phi_{\alpha 12}} & e^{i\phi_{\alpha 13}} \\ e^{i\phi_{\alpha 21}} & e^{i\phi_{\alpha 22}} & e^{i\phi_{\alpha 23}} \\ e^{i\phi_{\alpha 31}} & e^{i\phi_{\alpha 32}} & e^{i\phi_{\alpha 33}} \end{bmatrix}. \qquad (2.8)$$

In 1997, it was shown by Branco et al. that if suitable values are taken for $\phi_{\alpha ij}$, all the quark masses and the absolute values of the CKM matrix $|V_{CKM}|$ are correctly reproduced [4]. The part of its result is given as $m_u = 1.0\text{MeV}, m_c = 1.35\text{GeV}, m_t = 174\text{GeV}, m_d = 6.5\text{MeV}, m_s = 165\text{MeV}, m_b = 5.4\text{GeV}$, and

$$|V_{CKM}| = \begin{bmatrix} 0.9752 & 0.2213 & 0.0032 \\ 0.2210 & 0.9745 & 0.0391 \\ 0.0117 & 0.0375 & 0.9992 \end{bmatrix}. \qquad (2.9)$$

Then, the rephrasing invariant CP violation measure J is estimated as $J = 1.8 \times 10^{-7}$.

This is much smaller than the experimental value $J_{\text{exp.}} = 3.08 \times 10^{-5}$ [13]. It is hence clear that some new contribution to CP violation beyond the SM is necessary. It also remains as a problem why the top quark is much heavier than the bottom quark.

3. THDM within USY hypothesis

3-1. The model

There remain two problems in the SM within USY.

1. The rephrasing invariant CP violation measure J is much smaller than the experimental value.
2. It is not naturally explained why the top quark is much heavier than the bottom quark.

To solve these problems, we investigate the THDM within USY hypothesis, by taking the following two ansatz.

- The moduli of the Yukawa couplings are the same as c.
- The vev's of the two Higgs doublets have the same absolute value as v.

The Yukawa couplings are given as

$$\begin{aligned} L_{Yukawa} = &\lambda_{uij}{}^{(1)} q_{iL}{}^{\dagger} \Phi^{(1)} u_{jR} + \lambda_{uij}{}^{(2)} q_{iL}{}^{\dagger} \Phi^{(2)} u_{jR} \\ &\lambda_{dij}{}^{(1)} q_{iL}{}^{\dagger} \Phi^{(1)} d_{jR} + \lambda_{dij}{}^{(2)} q_{iL}{}^{\dagger} \Phi^{(2)} d_{jR} \qquad + h.c. \end{aligned} \tag{3.1}$$

where $\lambda_{\alpha ij}{}^{(x)} = c\exp(i\delta_{\alpha}{}^{(x)}) \cdot \exp(i\phi_{\alpha ij}{}^{(x)})$ and $q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$. Then, the quark mass matrices are given as

$$M_{uij} = \frac{cv}{\sqrt{2}} \left[\exp(i\delta_u{}^{(1)}) \cdot \exp(i\phi_{uij}{}^{(1)}) + \exp(i\delta_u{}^{(2)}) \cdot \exp(i\phi_{uij}{}^{(2)}) \right] \tag{3.2}$$

$$M_{dij} = \frac{cv}{\sqrt{2}} \left[\exp(i\delta_d{}^{(1)}) \cdot \exp(i\phi_{dij}{}^{(1)}) + \exp(i\delta_d{}^{(2)}) \cdot \exp(i\phi_{dij}{}^{(2)}) \right] \tag{3.3}$$

It is supposed here that the deviation from the flavor democracy is small as $\phi_{\alpha ij}{}^{(x)} \ll 1$.

For simplicity we begin to study in the limit $m_u = m_d = m_c = m_s$ with the democratic mass matrices,

$$M_u = \frac{cv}{\sqrt{2}} \left(\exp(i\delta_u{}^{(1)}) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \exp(i\delta_u{}^{(2)}) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) \tag{3.4}$$

$$M_d = \frac{cv}{\sqrt{2}}\left(\exp(i\delta_d^{(1)})\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \exp(i\delta_d^{(2)})\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) \tag{3.5}$$

In this limit the top-bottom hierarchy $m_b \ll m_t$ is derived only when $\delta_u^{(1)}$ is almost the same as $\delta_u^{(2)}$, and the difference between $\delta_d^{(1)}$ and $\delta_d^{(2)}$ is nearly π.

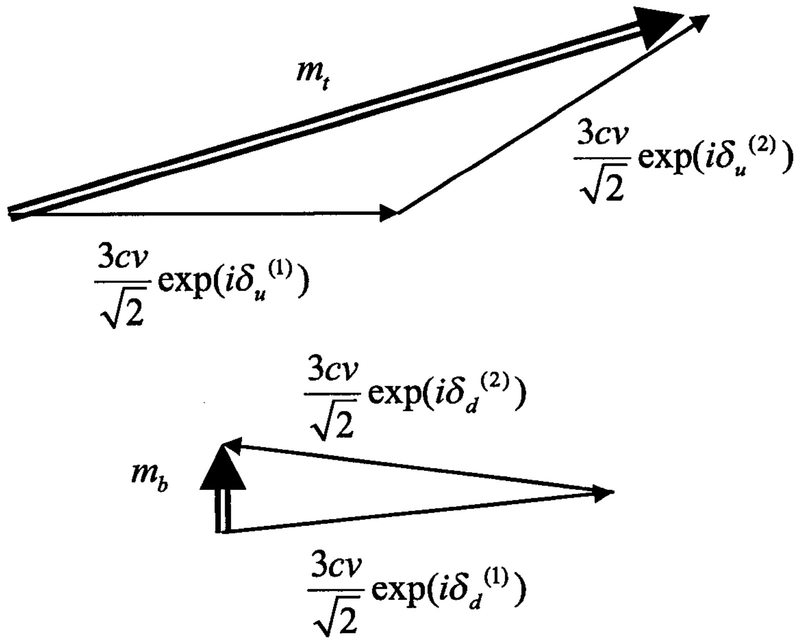

Fig1. Representation of $m_b \ll m_t$.

By perturbative calculation we found the phases $\phi_{\alpha ij}^{(x)}$ to reproduce the experimental values of the V_{CKM} and the rephrasing invariant CP violation measure J. The CKM matrix may be represented with the Wolfenstein parameterization as

$$V_{CKM} = \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}. \tag{3.6}$$

For simplicity it is supposed that $\delta_u^{(1)} = \delta_u^{(2)}$,and $\nu \simeq 29$. Then, the relations for the phases $\delta_d^{(1)}, \delta_d^{(2)}$,and $\phi_{\alpha ij}^{(x)}$'s are found in terms of the Wolfenstein parameters as

$$\left|\delta_d^{(1)} - \delta_d^{(2)}\right| \simeq \pi - \frac{m_b(1-2\lambda)}{3\nu}, \ \phi_{uij}^{(1)} \simeq 0, \ \phi_{uij}^{(2)} \simeq 0,$$

$$\phi_{d11}^{(1)} \simeq \phi_{d11}^{(2)} \simeq 0, \ \phi_{d22}^{(1)} \simeq \phi_{d22}^{(2)} \simeq \frac{2m_b\lambda}{3\nu}, \ \phi_{d33}^{(1)} \simeq \phi_{d33}^{(2)} \simeq \frac{m_b\lambda}{3\nu},$$

$$\phi_{d12}^{(1)} \simeq \phi_{d21}^{(2)} \simeq \frac{m_b\lambda[2 - A\eta\lambda^2(1+\lambda)]}{6\nu}, \ \phi_{d21}^{(1)} \simeq \phi_{d12}^{(2)} \simeq \frac{m_b\lambda[2 + A\eta\lambda^2(1+\lambda)]}{6\nu},$$

$$\phi_{d13}^{(1)} \simeq \phi_{d31}^{(2)} \simeq \frac{m_b\lambda[1 - A\eta\lambda^3]}{6\nu}, \ \phi_{d31}^{(1)} \simeq \phi_{d13}^{(2)} \simeq \frac{m_b\lambda[1 + A\eta\lambda^3]}{6\nu},$$

$$\phi_{d23}^{(1)} \simeq \phi_{d32}^{(2)} \simeq \frac{m_b\lambda[3 + A\eta\lambda^2(1+\lambda)]}{6\nu}, \ \phi_{d32}^{(1)} \simeq \phi_{d23}^{(2)} \simeq \frac{m_b\lambda[3 - A\eta\lambda^2(1+\lambda)]}{6\nu}. \tag{3.7}$$

3-2 Scalar mediated FCNC's

This THDM provides FCNC's as follows.

- Scalar mediated FCNC's are present at the tree level.
- Gauge boson Z mediated FCNC's are absent at the tree level.

On the perturbative calculation, the scalar mediated FCNC's appear to be too large to satisfy experimental value in the case that the nondiagonal elements of the Yukawa couplings are of the same order as the diagonal ones. In order to suppress the FCNC's sufficiently, it will be necessary that the first and the second terms in Eq. (3.1) are almost simultaneously diagonalized by the unitary transformations U_{uL} , and U_{uR} to diagonalize the up-type quark matrix M_u , while the third and the fourth terms in Eq. (3.1) are almost simultaneously diagonalized by the unitary transformations U_{dL} , and U_{dR} . to diagonalize the down-type quark mass matrix M_d . Hence, the phases $\phi_{dij}^{(x)}$ to break the democracy should be much smaller than the phase difference $\left|\delta_d^{(1)} - \delta_d^{(2)}\right|$. It is a future task to find the allowed range of $\phi_{\alpha ij}^{(x)}$ for estimating the possible magnitude of the scalar mediated FCNC's.

4. Summary

The THDM within USY hypothesis is fascinating as a possible solution to the flavor problem in the SM. It is shown that the quark masses, V_{CKM}, and rephrasing invariant CP violation measure J can all be reproduced. Moreover, the scalar mediated FCNC's appear at the tree-level, while the Z boson mediated ones do not. So far, suitable values of $\phi_{\alpha ij}^{(x)}$'s are not found yet so as to suppresses sufficiently the scalar mediated FCNC's satisfying the experimental data. This will be deserved for a future investigation.

Acknowledgement

We would like to thank Kobe Kaisei college for providing some reference books.

References

[1] S.L.Glashow,Nucl.Phys.22(1961)579;
S.Weinberg,Phys.Rev.19(1967)1264;
A.Salam,Elementary Particle Physics,ed. By N.Svartholm
(Almgvist and Willsells,Stockholm, 1968)367;
W.Marciano and H.Pagels, Phys.Rep.36(1978)137.

[2] G.C.Branco and J.I.Silva-Marcos, Phys.Lett.B237(1990)446.

[3] G.C.Branco and J.I.Silva-Marcos, Phys.Lett.B359(1995)166.

[4] G.C.Branco, D.Emmanuel and J.I.Silva-Marcos, Phys.Rev.D56(1997)107.

[5] I.S.Sogami,H.Tanaka and T.Shinohara, Prog.Theor.Phys.101 (1999) 707.

[6] T.Shinohara,H.Tanaka and I.S.Sogami, Prog.Theor.Phys.100 (1998) 615.

[7] J.I.Silva-Marcos, Phys.Lett.B443(1998)276.

[8] J.I.Silva-Marcos, hep-ph/0102079,February 2001.

[9] G.C.Branco, M.N.Rebelo and J.I.Silva-Marcos, hep-ph/9802340,February 1998.

[10] J.I.Silva-Marcos,Phys.Rev.D59(1999)091301.

[11] J.I.Silva-Marcos, Phys.Rev.D62 (2000) 073004.

[12] K.Higuchi, M.Senami and K.Yamamoto Kobe Kaisei.Rev.41,139(2003);
K. Higuchi, M. Senami and K. Yamamoto, Phys. Lett. B638, 492 (2006).

[13] See http://pdg.lbl.gov/ (Particle Data Group).